ABSTRACT

thesis of Altybay Arshyn
on the topic: **«Development of high-performance parallel algorithms and software complex for modeling hyperbolic type equations with singular coefficients: tsunami and acoustic wave propagation»**
submitted for the degree of Doctor of Philosophy (PhD) in the specialty
6D075100 – « Computer Science, Computer Engineering and Management»

Relevance of the research topic. At present, randomly occurring and rapidly changing processes leads to huge environmental and economic problems. Therefore, the modeling of such processes is very important. Many such problems are modeled by hyperbolic type equations with singular coefficients.

If we consider the worst tsunami in recent history the hardest event occurred on December 26, 2004, a magnitude 9.1 earthquake struck northern Indonesia, affecting 14 countries in the Indian Ocean and killing about 230,000 people. July 17, 2006, a magnitude 7.7 earthquake struck off the town of Pangandaran and set off a tsunami of 2 m high which had killed more than 300 people. March 11, 2011, a magnitude 9.0 earthquake near Tōhoku caused a great tsunami struck and killed more than 18000 people.

In the Caspian Sea, earthquakes have been very frequent in recent years, for example, 10 earthquakes in 2016, 12 earthquakes in 2017, the last earthquake in February 2020. According to historical data, in 957 an earthquake shook the Caspian Sea and destroyed 15 settlements in the Iranian region. On May 14, 1970, as a result of an earthquake in the Buinak region near the epicenter of Makhachkala, a tsunami occurred in the Caspian Sea, as a result of which 20 settlements were flooded and areas of Makhachkala were flooded, 31 people died and 45,000 were left homeless. At the beginning of 2000, the Russian Institute of Oceanology published a brochure entitled “Tsunamis in the Caspian Sea”, which predicts a possible tsunami wave height of 3 meters in the Caspian and Black Seas. Therefore, modeling and forecasting tsunami waves on the coast is very important.

We are allowing h to be a (positive) distribution, for example, allowing the case $h = 1 + \delta$, involving the δ-distribution. Such type of setting appears in applications, for example when one is looking at the behaviour of a particle in irregular electromagnetic fields: in the case of Landau Hamiltonian on $\mathbb{R}^n$, and the corresponding wave equation was analysed by the authors in [6]. While from the physical point of view (of irregular electromagnetic fields) such situation is natural and one expects the well-posedness, mathematically the equation is difficult to handle because of the general impossibility to multiply distributions (recall the famous Schwartz impossibility result from [80]).

Here if the singular coefficients are delta like function, then there is no classical solution. To deal with such problems we use the concept of very weak solutions [5-8].

The simulation of physical processes mentioned above on a large scale and for a long time requires large computational costs. If the computational algorithm

is sequential, then the computational costs are even larger. A temporary solution to avoid this problem is parallelization.

Efficiently parallelizing numerical methods and algorithms on a multicore processor was born in 2004 due to the fact that the physical limit forced the use of more processors on a silicon crystal.

Many engineering and scientific applications often require the simultaneous solution of a large number of equations with variable coefficients. The primary aim of this thesis work is to take advantage of the computational power of various modern parallel processor architectures to accelerate the computational speed of some mathematical problems by giving new algorithms and solutions.

The purpose of the dissertation work. Development of high-performance parallel algorithms and software complex for numerical solutions of hyperbolic type equations with singular coefficients such as tsunami and acoustic wave equation.

Research objectives realizing the goal of the dissertation work:

1) To design and analyze finite difference schemes for the 1D and 2D hyperbolic type equations, and to elaborate and study the implicit finite difference scheme for our equations;

2) To solve numerically the tsunami and acoustic wave equation in one and two dimensions by using the implicit finite difference scheme;

3) To parallelize the sequential algorithms using CUDA and MPI technologies;

4) To develop software complex for the investigation of the wave equation with singular coefficients.

The object of study. High-performance parallel computing, numerical methods, parallel programming technologies, hyperbolic type partial differential equations with singular coefficients, finite difference schemes, software application design tools.

The subject of study. Numerical analysis, numerical methods, parallel numerical algorithms for solving tridiagonal systems, software development technologies.

Scientific novelty. Proof of the existence, uniqueness and consistency of very weak solutions of the tsunami equation and justification by numerical modeling.

Development of parallel algorithm for the numerical solution of the two-dimensional wave equation with a singular coefficient using the MPI technology based on an implicit difference scheme.

Development of parallel algorithm for the numerical solution of the two-dimensional tsunami equation using the CUDA technology based on an implicit difference scheme.

Development of parallel hybrid algorithm for the numerical solution of a two-dimensional acoustics wave equation based on an implicit difference scheme.

Development of open-source, cross-platform software complex for numerical solution and investigation of hyperbolic type equation with singular coefficients.

The main provision for the defense

- Proof of the existence, uniqueness and consistency of very weak solutions of the tsunami equation and justification by numerical simulations.
- The developed parallel computational algorithm for the numerical solution of the two-dimensional wave equation with singular coefficients.
- The developed parallel algorithm for the numerical solution of the two-dimensional tsunami equation using the CUDA technology.
- The developed parallel hybrid algorithm for the numerical solution of the two-dimensional acoustics wave equation.
- The developed software complex for investigation of hyperbolic type equations with singular coefficients.

The theoretical significance of this work lies on the existence, uniqueness and consistency of very weak solutions to the tsunami equation and justified by numerical simulations.

The practical significance of the work is as follows:

The developed parallel algorithms for the numerical solution of hyperbolic equations with singular coefficients are applied to simulate a tsunami in the Caspian Sea; Developed software can be used to study waves in heterogeneous media in various fields of science.

Volume and structure of work. The thesis consists of an introduction, 3 sections and a conclusion, a list of references and an appendix. The total volume of the thesis is 99 pages, 40 figures, 8 tables. The list of references consists of 89 titles.

In the introduction, the relevance of the topic of the dissertation work, goals, as well as tasks for achieving this goal are discussed. The results obtained so far, their scientific novelty and significance are described.

In the first chapter, we present the mathematical models of hyperbolic type equations with singular coefficients such as tsunami and acoustic wave equation. Then we transform the partial differential equations to finite difference schemes. We compare explicit and implicit finite difference schemes, as a result of comparing we choose implicit scheme for our further implementation. In this chapter, we also theoretically prove that the existence, uniqueness and consistency of very weak solutions to the tsunami equation and justify it by numerical modeling.

At the end of this chapter, we will use our model to study a tsunami in the Caspian Sea, perform numerical simulations, and make various predictions of how high the tsunami will reach the coast, depending on the height of the first wave.

In the second chapter, we consider parallel numerical implementation of hyperbolic type wave equations. Firstly, we present MPI implementation of 2D wave equation with a distributional coefficient then CUDA implementation of 2D tsunami wave equation and related computational results.

At the end of this chapter, we present a hybrid implementation of acoustic wave problem then we compare the results from the different implementations.

In a hybrid implementation, joint use of Open MP, CUDA and MPI technologies to solve one problem, the result of the calculations shows that this implementation gives very good results.

In the third chapter, we describe the software complex for investigation of wave equation with singular coefficients. This software is open-source, cross-platform, and written in, one of the modern programming languages Python. This software will help researchers who investigate hyperbolic type equations with singular coefficients.

In the conclusion, conclusions of this dissertation work are presented.

Confidence level and validation results. The results of the study were discussed at scientific seminars of the Department of Computer science of the al-Farabi Kazakh National University and were reported at the following international conferences:

1. International Scientific Conference of Students and Young Scientists «Farabi alemi», Almaty, Kazakhstan, April 10-12, 2018
2. XLII Международной научно-практической конференции на тему: «Инновационные технологии на транспорте: образование, наука, практика» в рамках реализации Послания Президента РК Н. Назарбаева «Новые возможности развития в условиях четвертой промышленной революции», 18 апреля 2018 года
3. International Scientific Conference of Students and Young Scientists «Farabi alemi», *Almaty, Kazakhstan, April 6-9, 2020*

12 articles were published on the topic of the dissertation, the copyright certificate and act of implementation were received:

1. Altybay A., Ruzhansky M., Tokmagambetov N. Wave equation with distributional propagation speed and mass term: numerical simulations. // Applied Mathematics E-Notes. – 2019. – Vol. 19. – P. 552-562. (Scopus, Q3)
2. Altybay A., Ruzhansky M., Tokmagambetov N. A parallel hybrid implementation of the 2D acoustic wave equation // *International Journal of Nonlinear Sciences and Numerical Simulation*. – 2020. – Vol. 21, Iss. 7-8. – P. 821-827 (Scopus, Q2)
3. Altybay A., Ruzhansky M., Sebih M. E., Tokmagambetov N. Fractional Klein-Gordon equation with singular mass // *Chaos, Solitons & Fractals*, 2021. –Vol. 143. – P. 110579-110647. (Scopus, Q1)
4. Altybay A., Ruzhansky M., Sebih M. E., Tokmagambetov N The heat equation with strongly singular potentials // *Applied Mathematics and Computation*, – 2021. – Vol. 399. – P. 126-132. **(**Scopus, Q1**)**

5. Altybay A., Ruzhansky M., Sebih M. E., Tokmagambetov N. Fractional schrödinger equation with singular potentials // *Reports on Mathematical Physics*, . – 2021. – Vol. 87. №1. – P. 129-144 **(**Scopus**,** Q3**)**
6. Arshyn Altybay, Niyaz Tokmagambetov. On numerical simulations of the 1D wave equation with a distributional coefficient and sourse term. International Journal of Mathematics and Physics. Al-Farabi Kazakh national university. Volume 8, Number 2, 2017. ISSN 2218-7987.
7. Altybay A., Tokmagambetov N. A parallel algorithm for solving the two-dimensional wave equation with a singular coefficient // KazNTU Bulletin. – 2019. – Vol. 1. – P. 404-410.
8. Altybay A., Tokmagambetov N. MPI parallel implement of a wave equation using an implicit finite difference scheme. // KBTU Bulletin. – 2020. №1(52). – P. 112-120.
9. Altybay A., Tokmagambetov N. GPU computing for 2d wave equation based on implicit finite difference schemes // Bulletin NIA RK. – 2020. №3(77). – P. 32-42.
10. Altybay A. Numerical simulation of one hyperbolic type equation with a delta-like coefficient // International Scientific Conference of Students and Young Scientists «Farabi alemi», Almaty, Kazakhstan, – 2018. – P. 188-189.
11. Altybay A. On numerical simulations of the 1d wave equation with a distributional coefficient. comparison of the cases with neumann and dirichlet boundary conditions // XLII Международной научно-практической конференции на тему: «Инновационные технологии на транспорте: образование, наука, практика». – 2018. – Vol. 2. – P. 323-324.
12. Altybay A. Numerical simulation of tsunami equation and GPU computing // International Scientific Conference of Students and Young Scientists «Farabi alemi**»,** Almaty, Kazakhstan. – 2020. – P. 10-11.